

spot at the far ear? One would think so, if the near-ear level is a strongly monotonic function of direction—and the hearer has already acquired some sense of the absolute loudness of the tones from the experiment's repeated trials.

The measured far- and near-ear levels are shown in figure 2c for the same listener as in 2b. Clearly, the shape of his ILD curve is determined mostly by his far-ear levels. The θ dependence of his near-ear level is weak and erratic. So it could do little to resolve the angular ambiguity in the far-ear levels.

The green curve in figure 2c shows the near-ear levels predicted by the spherical-head model. That calculated angular dependence is indeed monotonic, but it rises only a meager 3 dB from 0° to 90° . By contrast, two of the five subjects had strong monotonic near-ear rises of about 10 dB, and they consistently had the best test scores. But

like the others, they fell afoul of the ILD ambiguity more often than not, even when the separate single-ear levels could, in principle, have resolved it.

Finally, the team investigated the effects of timing cues added to the test tones. In one experiment, they imposed a 100-Hz amplitude modulation on the pure 1500-Hz tones. And in another experiment, they replaced the pure tones by spiky narrow-band noise centered on 1500 Hz. In both experiments, almost all the subjects did very well. With resolvable and unambiguous interaural timing differences now at their disposal, they seem to have largely ignored the misleading ILDs.

Interestingly, the one striking exception was one of the two subjects with the strongest near-ear monotonic level rises and the best scores in the earlier experiments. He gained nothing from the timing cues and continued to be

misled by the ILDs. It's as if his acoustic-level perception thought itself too good to be ignored, even in the presence of more reliable timing information.

That last result supports the idea, first suggested by experiments in the 1970s, that when conflicting localization cues arrive at a central processor in the nervous system, different hearers will weight them quite differently. "This idiosyncratic unconscious weighting is not necessarily hard-wired in early childhood," says Hartmann. "But failed attempts at retraining in those old experiments, as in our own, indicate that such things are very resistant to change."

Bertram Schwarzschild

Reference

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Holograms tie optical vortices in knots

Using techniques from mathematics and optical wavefront engineering, researchers demonstrate that lines of zero intensity in a light beam can be shaped into links and loops of arbitrary topology.

What's the physical significance of a knot? William Thomson (who later became Lord Kelvin) must have asked himself that question in 1867 when he proposed his "vortex atom" hypothesis—the idea that different knotted configurations of swirling vortex lines in the ether are manifest in nature as different atoms. Earlier that year he'd witnessed Peter Guthrie Tait's intriguing experiments on smoke rings. And

based on earlier work by Hermann von Helmholtz, he knew that lines of vorticity are topologically conserved quantities. The closed loop and knotted configurations Thomson envisioned would thus be discrete and immutable—ideal traits for a theory of atoms.

Although initially skeptical, Tait came to believe that one could account for the rich variety of atoms in the periodic table by systematically building a

classification system for all types of knots. For the catalog he produced, Tait is now viewed as the father of knot theory. Thomson, for his part, had to abandon the vortex atom hypothesis. Ironically, though, it wasn't due to the nonexistence of the luminiferous ether—the 19th century's perfect fluid—but to an inability to analytically prove the temporal stability of a vortex knot.

Still, the subsequent history of

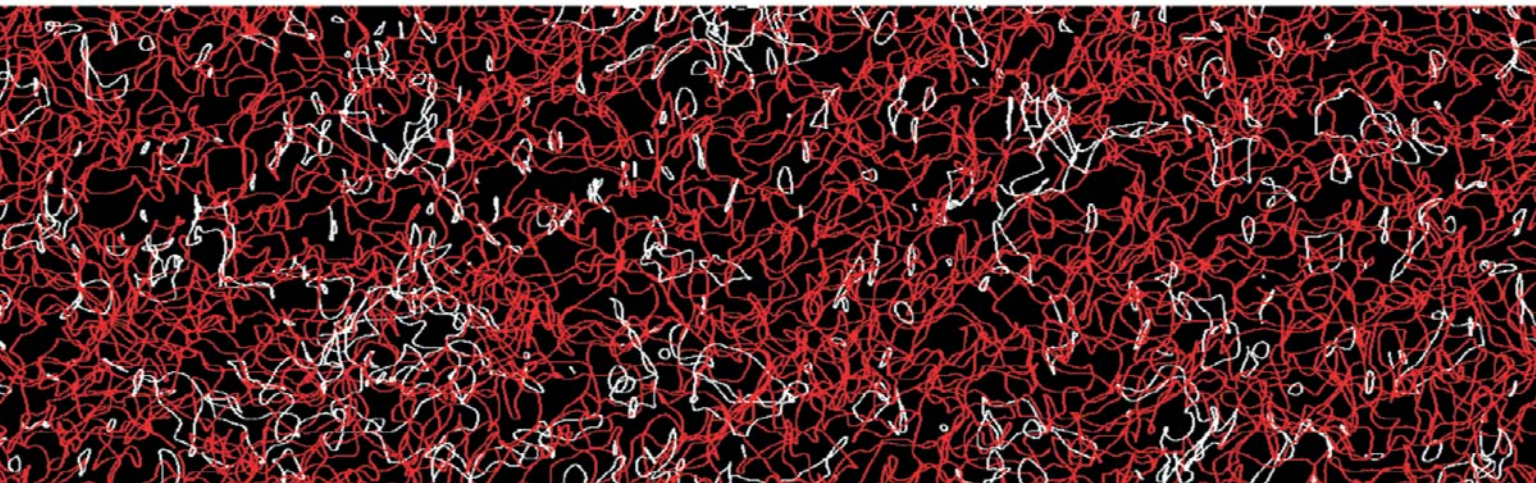


Figure 1. Speckleghetti. When coherent light from a laser scatters off a rough surface, it evinces a mottled speckle pattern of light and dark spots on that surface. The interference pattern is not restricted to a two-dimensional plane, though. In this numerical simulation of 625 randomly superposed plane waves propagating from left to right, points of perfectly destructive interference in the 2D plane transverse to the beam extend into tangled lines (red) and loops (white), known as optical vortices, in the 3D volume of a light beam. (Adapted from ref. 2.)

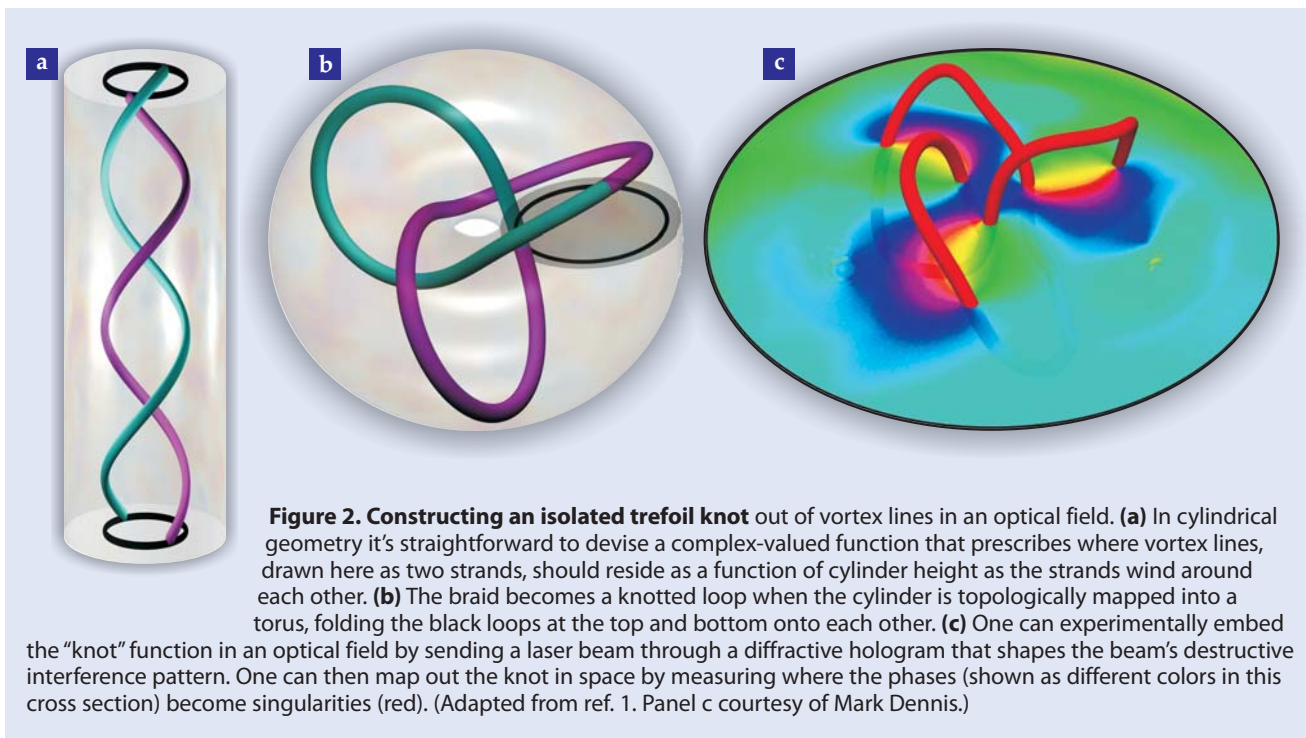


Figure 2. Constructing an isolated trefoil knot out of vortex lines in an optical field. **(a)** In cylindrical geometry it's straightforward to devise a complex-valued function that prescribes where vortex lines, drawn here as two strands, should reside as a function of cylinder height as the strands wind around each other. **(b)** The braid becomes a knotted loop when the cylinder is topologically mapped into a torus, folding the black loops at the top and bottom onto each other. **(c)** One can experimentally embed the "knot" function in an optical field by sending a laser beam through a diffractive hologram that shapes the beam's destructive interference pattern. One can then map out the knot in space by measuring where the phases (shown as different colors in this cross section) become singularities (red). (Adapted from ref. 1. Panel c courtesy of Mark Dennis.)

physics is filled with examples of systems—among them cold turbulent superfluids, hot magnetized plasmas, and cosmic strings in the evolving universe—in which the knottedness of vortex lines is thought to play a prominent role. In their efforts over the past few years to quantify and understand the shared topological properties of such disparate systems, Mark Dennis (University of Bristol), Miles Padgett (University of Glasgow), and their UK colleagues have been exploring the topology of yet another system: a beam of light. The researchers have now demonstrated, using a sophisticated holographic-grating scheme, that the dark threads in a complex optical field can be shaped into loops, linked loops, and knots of arbitrary topology.¹ The trick lay in finding the exotic solutions to Maxwell's equations—or, more precisely, the paraxial wave equation—that do the job.

Speckle patterns

The physics of scattered waves underlies the achievement. When three or more optical waves interfere in space, complete destructive interference occurs along nodal lines. At those places, known as optical vortices or phase singularities, the light intensity is zero and the phase undefined. Circling them, the phase increases or decreases by 2π .

In a three-dimensional light field—such as laser speckle, observed when coherent light scatters from a rough sur-

face—the nodal lines can form the sort of complicated tangle shown in figure 1. The numerical simulations of speckle patterns that Padgett, his graduate student Kevin O'Holleran, and Dennis made by superposing random plane waves reveal a complex electromagnetic field marked by a network of continuous and looped optical vortices that thread the entire volume of the light field.

From the simulation data, the researchers mapped the distribution of topological features and identified vortex loops that were threaded or unthreaded by vortex lines and loops that were linked with other loops. Not surprisingly, larger loops were more likely to be linked or threaded than smaller loops.² Earlier experiments in which a frosted glass screen was placed in front of a helium–neon laser beam bore similar results.³ Although links between two loops were found occasionally, no knots appeared in either real laser speckle or the simulations of random superpositions, probably because the chance of their formation is so low.

Designer knots

In 2001, well before the speckle studies, Dennis and his then thesis adviser Michael Berry predicted that vortex knots—or "knotted nothings," as they dubbed them—could be artificially manufactured using the superposition of a few laser beams with properly designed intensities and phases. The trou-

ble with an early prescription they conceived was that the knots it created would be inextricably connected to infinitely long vortex lines and loops in the system. The theoretical challenge was thus to construct a solution to the time-independent paraxial wave equation—the small-angle limit of the general wave equation—that specifies a scalar field containing nothing but the desired knot. The experimental challenge, in turn, was to imprint that solution onto an actual light wave.

Wrestling with the theoretical issues last year, Dennis and his student Robert King experienced a revelation. The pair realized they could exploit a mathematical operation known as Milnor mapping, which transforms a complex-valued scalar function that's simple to construct but doesn't contain a knot into a more complicated function that does. Figure 2 outlines the idea. To build a trefoil knot, for instance, they devised a function that describes a periodic braid—two nodal lines that spiral around each other up or down a cylinder. Under Milnor mapping, the top and bottom of the cylinder bend around to meet each other, so that the braid's two lines are spliced into a single knotted loop residing in a torus.

The lines cross three times in a trefoil knot, but more complicated topologies readily emerge by increasing the lines' crossing number or altering the trajectory they take—from circular to figure-8, say—as they're braided. The technique

is powerful: In 1923 topologist James Waddell Alexander proved that braiding yields every type of knot.

To experimentally create the scalar “knot” field, Padgett, along with students O’Holleran and Barry Jack, used a diffractive hologram as a spatial light modulator. The hologram, essentially a computer-controlled liquid-crystal film, generates the appropriate interference pattern, pixel by pixel, across an incoming laser beam’s profile. The light propagating from the hologram then has a phase pattern determined by the shape of its grating lines and an intensity determined by their contrast.

Typically, the dynamic range of intensity in the theoretically calculated knot cannot be achieved experimentally because of technical limitations of the hologram. So an iterative procedure that incrementally lowers the required contrast while preserving the knot was devised to optimize the hologram’s design.

Figure 2c shows the experimental trefoil knot, made evident by mapping the optical field’s phase distribution. In practice, the Glasgow team interferometrically measured the various phases, represented by different colors, across about 100 cross sections of the beam. The phase singularities, points where the colors meet in each cross sec-

tion, were then stitched together into lines that make up the knot.

Other waves, other knots

As linear solutions to a time-independent wave equation in free space, the vortex-knot patterns are stable and static. But there also exist exotic time-dependent solutions of Maxwell’s equations that describe knotted structures. Two years ago physicists William Irvine (New York University) and Dirk Bouwmeester (University of California, Santa Barbara) studied solutions, based on a topological construction known as a Hopf fibration, in which all electric and magnetic field lines are closed loops and any two electric or magnetic field lines can be linked and knotted.⁴

Because they’re scalar, vortices are relatively easy to manipulate using holograms. In contrast, Irvine and Bouwmeester’s field-line solutions have the vector character of the electric and magnetic fields built into them, presenting the experimental challenge of dynamically measuring their vectors in 3D space.

Knot theory has already influenced particular areas of physics—most notably topological quantum computing (see the article by Sankar Das Sarma, Michael Freedman, and Chetan Nayak in PHYSICS TODAY, July 2006, page 32).

The exquisite control that researchers can now exert to knot optical fields may catalyze other applications. The light around optical vortices has orbital angular momentum and thus can exert a torque, which has been widely exploited in ringlike optical traps, for example (see the article by Padgett, Johannes Courtial, and Les Allen in PHYSICS TODAY, May 2004, page 35). Padgett speculates that the rarity and sensitivity of optical knots within scattered light may even form the basis for a new kind of molecular sensor.

Applications aside, Irvine finds them fascinating examples of the close connection between geometry and Maxwell’s equations. “I can’t say whether knots tied in fields are important physical objects or not,” he says. “It probably depends on what exactly is tied up. But for me, they’re curious, beautiful, and hard to resist.”

Mark Wilson

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Model sheds light on the language of color

The process by which human societies collectively decide which segments of the visible spectrum get their own names can be studied by computer simulation.

Speakers of the same language generally agree whether two hues are different shades of the same color, in the sense that reddish orange and yellowish orange are both orange, or whether they are entirely different colors. And although speakers of different languages (especially pairs of languages whose speakers don’t interact) often disagree about what constitutes “the same color,” the disagreement isn’t as great as would be expected by chance. Statistical analysis of the World Color Survey’s data set—a collection of color categories in 110 languages from nonindustrial populations—found that the languages clustered together in color space to a greater degree than did sets of randomly generated categories.¹

Researchers have used several computational approaches to understand how languages’ color categories develop. Among them is the work of Andrea Baronchelli, a physicist at the Polytechnic University of Catalonia in Barcelona, Spain, and his collaborators,

